CHARACTERISTIC BEHAVIOR WHEN SURFACE HEAT PULSE ACTS ON METAL

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The powerful thermal effect exerted on a metal by a heat pulse is considered as a function of the pulse introduction time, for fixed surface density.

A powerful thermal action on metal will excite vigorous surface evaporation, with rapid motion of the evaporation front into the metal. When a moving coordinate system associated with the evaporation front is employed, a satisfactory approximation to the process occurring under such conditions is given by the following formulation of the problem [1]:

$$\frac{\partial T(x,\ t)}{\partial t} = a\frac{\partial^2 T(x,\ t)}{\partial x^2} + v_0 \exp\left[-\frac{T_m}{T(0,\ t)}\right]\frac{\partial T(x,\ t)}{\partial x};$$
$$\frac{\partial T(\infty,\ t)}{\partial x} = 0;\quad -\frac{\partial T(0,\ t)}{\partial x} = (ac_V)^{-1}\left\{F - v_0 r_V \exp\left[-\frac{T_m}{T(0,\ t)}\right]\right\}, \tag{1}$$
$$0 \leqslant x \leqslant \infty;\quad 0 \leqslant t \leqslant t_{\mathrm{n}};\quad T(x,\ 0) = T_0.$$

Computer solution of this essentially nonlinear problem has shown that the process splits into two stages: transient and steady-state [1, 2]. In the steady mode, the process is governed by the following equations [1]:

$$F = v[r_V + c_V(T - T_0)] = v_0(Ay + B)/y \exp y, \tag{2}$$

$$x_{\mathrm{m}} = \frac{a}{v}\ln\frac{T - T_0}{T_{\mathrm{m}} - T_0} = \frac{a \exp y}{v_0}\ln\frac{T_m - T_0 y}{(T_{\mathrm{m}} - T_0)y}, \tag{3}$$

where

$$y = T_m/T;\quad v = v_0\exp(-y);\quad A = r_V - c_V T_0;\quad B = c_V T_m. \tag{4}$$

The duration of the transient stage lies within the estimate [2]

$$t_0 = a/v^2 = (a/v_0^2)\exp 2y, \tag{5}$$

while the increase in evaporation-front velocity during the transient can be approximated satisfactorily as

$$v(t) = v\,\mathrm{erf}(2t/t_0), \tag{6}$$

which yields the steady velocity value when $t \geq t_0$.

If the heat pulse is specified by the restricted surface density

$$W = Ft, \tag{7}$$

then when W = const we find a very unique relationship between the path traversed by the evaporation front,

$$x'_{\mathrm{ev}} = \int_0^t v(t)\,dt \tag{8}$$

and the melting isotherm,

$$x'_{\mathrm{m}} = x'_{\mathrm{ev}} + x_{\mathrm{m}} \tag{9}$$

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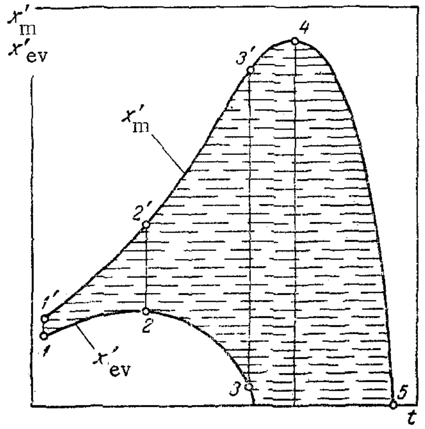

Fig. 1. General view of the relationship between the motion of the evaporation front (x'_{ev}) and the melting isotherm (x'_m) and time at which the pulse was introduced into the melting metal for a specified restricted surface density energy.

and the time at which the pulse was introduced. The machine-solution data [1] have been used to represent this relationship in general form (Fig. 1); we have shown several of the most characteristic points 1, 1', 2, 2', 3, 3', 4, and 5. Knowing their coordinates, we can satisfactorily construct the entire figure for any specified value of W. Each of these points corresponds to quite definite thermal-effect modes, which we shall henceforth treat as being the most characteristic. Let us look at these points separately.

Point 1. For a very high front velocity, the problem (1) becomes approximate [3], so that it is desirable to restrict v_1 to some value substantially less than the speed of sound, such as $v_1 = 10^{-1}v_0$, which corresponds to

$$F_1 = v_0 (A \ln 10 + B)/\ln 10 \exp(\ln 10);\quad t_{01} = 10^2 a/v_0^2. \tag{10}$$

For values of W that lie within the range of practical interest, 10^3-10^8 J/m^2, the time for which the pulse acts,

$$t_1 = W/F_1 = \ln 10 \exp(\ln 10)\, W/v_0 (A \ln 10 + B) \tag{11}$$

considerably exceeds the transient time t_{01}, which makes it possible to determine the path

$$x'_{1\,ev} = v_1 t_1 = \ln 10 \exp(\ln 10)\, W/10 (A \ln 10 + B) \tag{12}$$

traversed by the front without regard to the duration of the transient. In such a case, the coordinates of point 1 are uniquely determined by (11) and (12).

Point 1'. For this point, it is sufficient to determine the depression in the melting isotherm with respect to the front on the basis of the steady-mode conditions (3):

$$x_m = \frac{10\,a}{v_0} \ln \frac{(T_m/\ln 10) - T_0}{T_m - T_0}, \tag{13}$$

since F'_1 and t'_1 have the same values as for point 1, i.e., $F'_1 = F_1$ and $t'_1 = t_1$. Thus one of the coordinates of 1' is found from the expression

$$x'_{1m} = x'_{1ev} + x_m \tag{14}$$

in accordance with (12), (13), while the other is found from (11).

Point 2. If for W = const, the time for which the pulse acts increases from t_1 to t_2, this entails a reduction in F, v, and T from the values F_1, v_1, and T_1 to the values F_2, v_2, and T_2. In addition, the duration of the transient rises from t_{01} to t_{02}. Let $t_2 > t_{02}$, i.e., assume that the process has entered the steady phase. In this case, in accordance with (6) and (8), we can write

$$x'_{2\,ev} = \int_0^{t_{02}} v_2 \operatorname{erf}(2t/t_{02})\, dt + \int_{t_{02}}^{t_2} v_2 dt, \tag{15}$$

where it is no longer possible to neglect the duration of the transient. Calculation of the first term yields $0.875 v_2 t_{02}$; then from (15) we have

$$x'_{2\,ev} = v_2 (t_2 - 0.125\, t_{02}). \tag{16}$$

Now since $v_2 = v_0 \exp(-y_2)$, $t_{02} = a/v_2^2 = a/v_0^2 \exp 2y_2$, and in accordance with (2), (7)

$$t_2 = W/F_2 = W v_0 (A y_2 + B)/y_2 \exp y_2, \tag{17}$$

we can write (16) as

$$x'_{2\,ev} = [W y_2/(A y_2 + B)] - CB \exp y_2, \tag{18}$$

where C is determined by the constants of the metal

$$C = 0.125\, a/v_0 c_V T_m. \tag{19}$$

TABLE 1. Computational Results for Certain Metals, W = 1 J/mm^2

Metal	t, sec				x'_{ev}, m			x_m, m			P, bar			
	t_1	t_2	$t_3=t_4$	t_5	x'_1	x'_2	$x'_3=x'_4$	x_1	x_2	$x_3=x_4$	P_1	P_2	$P_3=P_4$	P_5
Al	$3,88\cdot10^{-8}$	$5,58\cdot10^{-6}$	$6,05\cdot10^{-4}$	$3,83\cdot10^{-3}$	$1,98\cdot10^{-5}$	$2,82\cdot10^{-5}$	$1,14\cdot10^{-6}$	$2,92\cdot10^{-7}$	$1,41\cdot10^{-5}$	$1,60\cdot10^{-4}$	$4,90\cdot10^{4}$	$3,54\cdot10^{2}$	$8,74\cdot10^{0}$	$8,83\cdot10^{-5}$
Ni	$1,64\cdot10^{-8}$	$1,33\cdot10^{-5}$	$2,20\cdot10^{-4}$	$1,16\cdot10^{-3}$	$8,20\cdot10^{-6}$	$3,26\cdot10^{-6}$	$7,10\cdot10^{-6}$	$5,54\cdot10^{-8}$	$4,67\cdot10^{-6}$	$4,77\cdot10^{-5}$	$1,21\cdot10^{5}$	$6,75\cdot10^{2}$	$1,10\cdot10^{2}$	$5,53\cdot10^{-3}$
Cu	$1,64\cdot10^{-8}$	$1,03\cdot10^{-5}$	$8,72\cdot10^{-5}$	$5,57\cdot10^{-4}$	$9,32\cdot10^{-6}$	$1,82\cdot10^{-5}$	$5,76\cdot10^{-7}$	$5,42\cdot10^{-7}$	$6,21\cdot10^{-5}$	$8,06\cdot10^{-5}$	$8,65\cdot10^{4}$	$1,07\cdot10^{2}$	$3,05\cdot10^{0}$	$1,02\cdot10^{-5}$
Zn	$9,18\cdot10^{-8}$	$2,95\cdot10^{-5}$	$1,57\cdot10^{-3}$	$1,01\cdot10^{-2}$	$3,36\cdot10^{-5}$	$5,21\cdot10^{-5}$	$4,32\cdot10^{-6}$	$1,71\cdot10^{-7}$	$1,29\cdot10^{-5}$	$2,04\cdot10^{-4}$	$5,84\cdot10^{4}$	$8,16\cdot10^{1}$	$7,12\cdot10^{1}$	$1,48\cdot10^{-1}$
Ag	$6,25\cdot10^{-8}$	$1,52\cdot10^{-5}$	$1,02\cdot10^{-3}$	$6,40\cdot10^{-3}$	$1,67\cdot10^{-5}$	$2,64\cdot10^{-5}$	$4,35\cdot10^{-6}$	$1,47\cdot10^{-7}$	$9,90\cdot10^{-6}$	$1,17\cdot10^{-4}$	$4,80\cdot10^{4}$	$9,40\cdot10^{1}$	$8,76\cdot10^{0}$	$2,25\cdot10^{-3}$
Cd	$1,90\cdot10^{-7}$	$8,63\cdot10^{-5}$	$3,38\cdot10^{-3}$	$2,16\cdot10^{-2}$	$5,09\cdot10^{-5}$	$8,73\cdot10^{-5}$	$1,00\cdot10^{-8}$	$3,08\cdot10^{-7}$	$2,90\cdot10^{-5}$	$3,12\cdot10^{-4}$	$2,78\cdot10^{4}$	$4,70\cdot10^{1}$	$3,20\cdot10^{1}$	$1,70\cdot10^{-1}$
Sn	$7,77\cdot10^{-8}$	$1,71\cdot10^{-5}$	$1,04\cdot10^{-2}$	$6,60\cdot10^{-2}$	$2,07\cdot10^{-5}$	$3,34\cdot10^{-5}$	$1,02\cdot10^{-6}$	$3,86\cdot10^{-7}$	$3,92\cdot10^{-5}$	$4,94\cdot10^{-4}$	$9,75\cdot10^{4}$	$2,05\cdot10^{2}$	$5,35\cdot10^{-8}$	$6,23\cdot10^{-18}$
W	$1,18\cdot10^{-7}$	$6,36\cdot10^{-7}$	$2,17\cdot10^{-5}$	$1,38\cdot10^{-4}$	$5,10\cdot10^{-6}$	$7,24\cdot10^{-6}$	$3,20\cdot10^{-8}$	$2,43\cdot10^{-7}$	$4,65\cdot10^{-6}$	$2,90\cdot10^{-5}$	$2,02\cdot10^{5}$	$1,63\cdot10^{3}$	$2,48\cdot10^{2}$	$3,25\cdot10^{-3}$
Pb	$2,96\cdot10^{-7}$	$7,57\cdot10^{-5}$	$2,15\cdot10^{-2}$	$1,37\cdot10^{-1}$	$4,00\cdot10^{-5}$	$6,58\cdot10^{-5}$	$1,07\cdot10^{-6}$	$4,43\cdot10^{-7}$	$9,10\cdot10^{-5}$	$5,48\cdot10^{-4}$	$1,72\cdot10^{4}$	$2,95\cdot10^{1}$	$8,27\cdot10^{-3}$	$2,46\cdot10^{-9}$

It follows from (18) that the path traversed by the front depends on $y_2 = T_m/T_2$, and for a certain value of y_2, $x'_{2\,ev}$ will reach a maximum. Maximizing (18), we find

$$C(Ay_2 + B)^2 \exp y_2 = W. \qquad (20)$$

Thus the coordinates of Point 2 are determined by (17) and (18), where y_2 must first be computed from the transcendental equation (20).

Point 2'. From this point, we need only determine the dip in the melting isotherm with respect to the front, using the steady-mode conditions (3),

$$x_{2m} = \frac{a \exp y_2}{v_0} \ln \frac{T_m - T_0 y_2}{(T_m - T_0)\, y_2}, \qquad (21)$$

where y_2 is also first found from the transcendental equation (20). As a consequence, one of the coordinates of 2' is determined by the expression

$$x'_{2m} = x'_{2ev} + x_{2m} \qquad (22)$$

in accordance with (18), (21), and the second from (17).

Point 4. This point is observed when conditions are such that evaporation can be neglected. In this case, the problem (1) becomes a simpler, linear problem:

$$\frac{\partial T(x,\ t)}{\partial t} = a\,\frac{\partial^2 T(x,\ t)}{\partial x^2};$$

$$\frac{\partial T(\infty,\ t)}{\partial x} = 0; \quad -\frac{\partial T(0,\ t)}{\partial x} = (ac_V)^{-1}F, \qquad (23)$$

$$0 \leqslant x \leqslant \infty; \quad 0 \leqslant t \leqslant t_n; \quad T(x,\ 0) = T_0,$$

which has the familiar solution

$$T(x,\ t) = T_0 + (2F\sqrt{t}/c_V\sqrt{a})\ \mathrm{ierfc}\ (x/2\sqrt{at}), \qquad (24)$$

where ierfc u is a tabulated function [4].

For the given point, at time $t = t_4$, $x = x_{4m}$, $T(x_{4m},\ t_4) = T_m$, $F = F_4 = W/t_4$; then from (24) we have

$$\mathrm{ierfc}\,(x_{4m}/2\sqrt{at_4}) = c_V(T_m - T_0)\sqrt{at_4}/2W. \qquad (25)$$

For a given W, we can calculate x_{4m} from (25) as a time function, $x_{4m} = x_{4m}(t)$. We use the maximum value of this function to find the desired values x_{4m}, t_4, and F_4. The appropriate calculations show that the maximum occurs when

$$\frac{x_{4m}}{2\sqrt{at_4}} = 0.45; \quad \frac{c_V\,(T_m - T_0)\sqrt{at_4}}{2W} = 0.225, \qquad (26)$$

and thus we can find the coordinate of Point 4:

$$t_4 = 0.2\,W^2/ac_V^2\,(T_m - T_0)^2, \qquad (27)$$

$$x_{4m} = 0.4\,W/c_V\,(T_m - T_0). \qquad (28)$$

Point 3. The given point does not lie on the time axis t, but only approaches it asymptotically as t increases. This occasions a certain indeterminacy in the choice of a criterion for estimating its position. It is reasonable to require that $x'_{3ev} \ll x'_{2ev}$ and, in addition, that the time t_3 corresponding to this point forms but a negligible part of the transient duration t_{03}. On the basis of these conditions, as calculations show, t_3 approaches t_4, and thus it is best to take $t_3 = t_4$ and $F_3 = F_4$, which is equivalent to combining Point 3' with Point 4, and Point 3 with the projection of Point 4 on the time axis.

The steady mode is naturally not reached when $t_3 = t_4$, but in accordance with the data for this mode ($y_3 = T_m/T_3$), we can write

$$F_3 = v_0 (Ay_3 + B)/y_3 \exp y_3 \tag{29}$$

in accordance with (3). On the other hand, in accordance with (7) and (27), we can write

$$F_4 = W/t_4 = 5ac_V^2 (T_m - T_0)^2/W. \tag{30}$$

Since we have taken $F_3 = F_4$, it follows from (29) and (30) that

$$(Dy_3 \exp y_3)/(Ay_3 + B) = W, \tag{31}$$

where D is determined by the constants of the metal,

$$D = 5ac_V^2 (T_m - T_0)^2/v_0. \tag{32}$$

From the data for y_3, calculated from (31), we can determine the transient duration,

$$t_{03} = a/v_3^2 = (a \exp 2y_3)/v_0^2, \tag{33}$$

following which the steady mode can set in for a heat flux $F_3 = F_4$. We can now also estimate the fraction of this duration that is represented by t_3,

$$\alpha = \frac{t_3}{t_{03}} = \frac{t_4}{t_{03}} = 0{,}2 \left[\frac{v_0 W \exp(-y_3)}{ac_V (T_m - T_0)}\right]^4 . \tag{34}$$

As the calculations show, for various metals, α amounts to about 10^{-2} or less. In such case, we can write

$$x'_{3\text{ev}} = \int_0^{t_4} v_3 \operatorname{erf}(2t/t_{03})\, dt = \frac{8 \cdot 10^{-2}}{\sqrt{\pi}} \left(\frac{v_0}{a \exp y_3}\right)^3 \left[\frac{W}{c_V (T_m - T_0)}\right]^4 . \tag{35}$$

Thus $x'_{3\text{ev}} \ll x'_{4\text{ev}}$, and in addition, $x'_{3\text{ev}} \ll x_{4\text{m}}$, which corresponds to the criterion for determination of the coordinates of the given point.

Point 5. This point corresponds to the time t_5 at which, for a given W, the melting point $T(0, t_5) = T_m$ is reached at the surface of the metal. Letting $x = 0$ and $t = t_5$ in (24), we find the coordinate of the given point,

$$t_5 = 4W^2/\pi a c_V^2 (T_m - T_0)^2. \tag{36}$$

In none of these calculations have we allowed for the heat of fusion L_V. If we do, then Eq. (3), which expresses conservation of energy for the steady mode, will take the more general form

$$F = v[r_V + L_V + c_V (T - T_0)]. \tag{37}$$

Since $L_V \ll r_V$, allowance for it will not noticeably change the values computed above for Points 1, 2, and 3. The dip in the melting isotherm, i.e., the positions of Points 1', 2', 4, and 5, can be affected noticeably when we take L_V into account. Rigorous investigation of this problem requires formulation and solution of the Stefan problem; we can, however, consider only approximate estimates for the positions of these points, using the reduced melting point

$$T_m^* = T_m + L_V/c_V \tag{38}$$

in the calculations. There is such a way of allowing for the heat of fusion [5], and although it is not terribly accurate, the computed results represent better the actual process than when the heats of fusion are neglected. Thus the positions of 1', 2', 4, and 5 should be found from the reduced melting point, which can be done by replacing T_m by T_m^* in (13), (21), (27), (28), (31), (35), and (36).

The evaporating atoms, leaving the evaporation front at high speeds, exert a strong effect, which entails the appearance of considerable reactive pressure, whose values can be found from the following expression [6]:

$$P = P_0 [\exp(-y)/\sqrt{y}], \tag{39}$$

where

$$P_0 = 2\gamma N l \nu \sqrt{2m\varepsilon}/A^* \sqrt{\pi}. \tag{40}$$

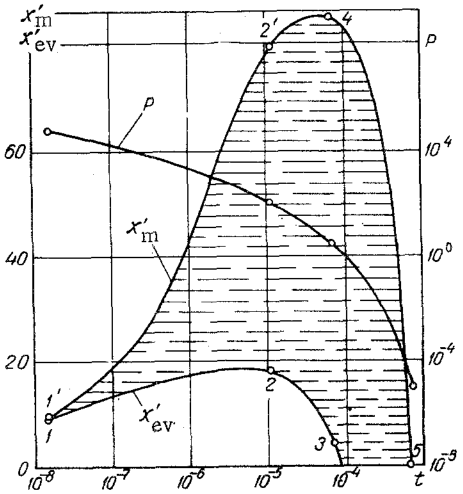

Fig. 2. Reactive pressure (P, bar), displacements of evaporation front (x'_{ev}, μ), and melting isotherms (x'_m, μ) as functions of the time (t, sec) at which a heat pulse is introduced into a metal (copper) for a fixed surface density W = 1 J/mm^2.

We need only consider the reactive pressure for the characteristic points discussed above, i.e., at time t_1, t_2, $t_3 = t_4$, and t_5, when $y = T_m/T(0, t)$ takes on the respective values

$$y_1 = T_m/T_1 = \ln 10, \tag{41}$$

$$y'_3 = y'_4 = T_m/T(0,\ t_4) = T_m/\{T_0 + [2(T_m - T_0)/\sqrt{0,\ 2\pi}]\}, \tag{42}$$

$$y_5 = T_m/T(0,\ t_5) = T_m/T_m, \tag{43}$$

while $y_2 = T_m/T_2$ is found from (20). Here y_1 and y_2 correspond to the steady-mode conditions, when y'_4 and y'_5 correspond to the final instant of introduction of the given pulse W for the nonsteady, transient stage of the process. This is important by virtue of the fact that in the steady phase, the pressures P_1 and P_2 act throughout almost the entire pulse, while for the nonsteady phase, P_4 and P_5 appear only at the end of the pulse.

Table 1 shows the computed results for the basic quantities at the characteristic points for certain metals. Figure 2 graphically illustrates the relationship between these quantities and the time for which the given impulse acts, in the case of copper.

As we see from Fig. 2, in the steady phase of the process, as the pulse introduction time increases, i.e., as we go from Point 1 to Point 2, the amount of liquid phase rises (x_m increases from 0.54 to 62.1 μ) while the reactive pressure drops sharply (from $8.65 \cdot 10^4$ to $1.07 \cdot 10^2$ bar). At Point 3, it has dropped to about 1 bar (~1 atm); it then drops to values that are of no practical importance. It is clear from the graph that the reactive pressure can become a very important factor in the evacuation of liquid metal, but only in the short-pulse region. As the pulse duration increases, there is an increase in the amount of liquid phase, and this is accompanied by a loss of reactive pressure.

NOTATION

c_V	is the specific bulk heat capacity;
r_V	is the specific bulk heat of evaporation;
v_0	is the speed of sound in metal;
L_V	is the specific bulk heat of fusion;
a	is the thermal diffusivity;
m, ν	are the mass and vibration frequency of the atoms of the metal, respectively;
N	is the Avogadro number and atomic weight;
ε	is the energy of the atomic bond;
T_m	is the characteristic temperature for the given metal, found from the condition $kT_m = \varepsilon$;
k	is the Boltzmann constant;
γ	is the density of the metal;
t	is the time for which the heat pulse acts;
F	is the density of the heat flux acting at the evaporation front.

LITERATURE CITED

1. A. G. Goloveiko, Inzh.-Fiz. Zh., 8, No. 2, 215 (1967).
2. A. G. Goloveiko, Izv. VUZ. Énergetika, No. 5, 79 (1968).
3. S. I. Anisimov, Teplofiz. Vys. Temp., No. 1, 116 (1968).
4. A. V. Lykov, The Theory of Heat Conduction [in Russian], GITTL (1952).
5. A. S. Zingerman, Fiz. Tverd. Tela, 1, No. 2 (1959).
6. A. G. Goloveiko, Izv. VUZ. Énergetika, No. 6, 83 (1966).